Testing the Lorentz and CPT Symmetry with CMB polarizations and a non-relativistic Maxwell Theory

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We present a model for a system involving a photon gauge field and a scalar field at quantum criticality in the frame of a Lifthitz-type non-relativistic Maxwell theory. We will show this model gives rise to Lorentz and CPT violation which leads to a frequency-dependent rotation of polarization plane of radiations, and so leaves potential signals on the cosmic microwave background temperature and polarization anisotropies.

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I. INTRODUCTION

The Lorentz and the CPT (Charge conjugation-Parity-Time reversal) symmetries are commonly assumed to be the most fundamental elements in constructing the models of elementary particle physics. Various attempts to break these symmetries have been intensively discussed in the literature. Recently motivated by the pioneering work in condensed matter physics[1], Hořava has proposed a class of models of non-relativistic quantum field theories [2, 3], in which the spatially higher derivative terms are introduced to improve their ultraviolet behaviors. This theory, when applied to the gravity sector, is power-counting renormalizable and hence potentially ultraviolet (UV) complete[4], although a strong coupling problem needs to be aware of[5]. Moreover, a class of non-relativistic Yang-Mills gauge theories in D+1 dimensions whose free-field limit exhibits quantum critical behavior with gapless excitations and dynamical critical exponent $z = 2$ have been studied in Ref. [2]. The class of these theories are expected to be engineered from the D-brane configurations of string theory[3], and in the large-N limit, they have weakly curved gravity duals. A general feature of this theory is that, the action does not need the usual diffeomorphism invariance of General Relativity, but has a fixed point in the infrared (IR) limit which can recover the Lorentz invariance as an accident symmetry. As a specific low energy scale presentation of these theories, a non-relativistic Maxwell theory appears to be the most potential one which might be detectable in experiments.

Probing for the violation of the Lorentz and the CPT symmetries is an important approach to searching for new physics beyond the standard model[6–10]. Up to now, however the Lorentz and the CPT symmetries have passed a number of high precision experimental tests in the ground-based laboratory and no definite signal of their violation has been observed [11]. So, the present Lorentz and the CPT violating effects, if they exist, should be very small to be amenable to the experimen-

tal limits. In the past several years, there has been a lot of studies on the lorentz and CPT test with the polarization of the cosmic microwave background radiation (CMB). One of the examples is to consider a scalar field which couples to the photon Chern-Simon term $\mathcal{L}_{\mathrm{int}} \sim \partial_\mu \phi A_\nu \widetilde{F}^{\mu\nu}$[6]. This interaction is Lorentz invariant, however breaks dynamically the CPT and lorentz symmetries during the evolution of the universe when ϕ rolls down along a potential and $\dot{\phi}$ gets a non-vanishing value. The parity violation, resulted by the rolling cosmological scalar[6] as well as the gravitational chirality[12], leads to the rotation of the CMB photon polarization when propagating in the universe[7, 13, 14], and the current CMB data mildly indicates a non-zero value of this rotation angle[15–18]. Importantly in comparison to the laboratory experiments, using the CMB polarization has been shown to be the most powerful approach to testing the Lorentz and CPT symmetries in the photon sector since the CMB photon has travelled a long time (almost the age of the universe).

In this letter we study the model for a system consisting of a scalar ϕ coupled to the photon field in a Lifshitz-type non-relativistic Maxwell theory and discuss its implications for CMB polarizations. Since the theory has different fixed points at UV and IR limits respectively, our results show that the rotation angle obtained in this model is quite different from what obtained in the usual case. For high energy photons, the rotation angle is frequency dependent and this dependence differs from that of Faraday rotation, which is a new feature of this model. For low energy photons, the rotation angle obtained in this model coincides with the one for a dynamical scalar field coupled to the Chern-Simons current in the $3 + 1$ dimensional spacetime.

This letter is organized as follows. In Section II we review very briefly on the CMB polarization and its connection to the CPT test. In Section III we review also very briefly on the construction of a Lifthitz-type theory. In Section IV we present in detail the model with $z = 2$. In Section V we study the implications of our model in CMB and calculate the rotation angle of the CMB pho-

ton. Section VI is our conclusion and discussions.

II. CMB POLARIZATION CONSTRAINTS ON CPT VIOLATION

In this section we review very briefly how to use the CMB information to constrain the CPT violation. In general it is convenient to use the Stokes parameters to study the CMB polarization. Considering a monochromatic electromagnetic wave of frequency ω propagating in the positive direction of z axis, which is described by an electric field $\vec{E}$, the Stokes parameters are defined as

$$
\begin{aligned}
I &\equiv \langle E_1 E_1^* \rangle + \langle E_2 E_2^* \rangle , \\
Q &\equiv \langle E_1 E_1^* \rangle - \langle E_2 E_2^* \rangle , \\
U &\equiv \langle E_1 E_2^* \rangle + \langle E_1^* E_2 \rangle , \\
V &\equiv i[\langle E_1 E_2^* \rangle - \langle E_1^* E_2 \rangle] ,
\end{aligned}
\tag{1}
$$

where the notation $\langle \rangle$ denotes the time average.

Among these parameters, Q and U reflect the CMB polarization which can be decomposed into a gradient-like (E) and a curl-like (B) component. For the standard theory of CMB, the TB and EB cross correlations vanish. However, with the presence of CPT violation by the interaction terms mentioned above, we could observe a rotation angle $\Delta\chi$ on the polarization vector of a photon[13, 19].

In the following we study how the rotation angle affects the CMB power spectra. In a flat FRW universe, we can expand the temperature and polarization anisotropies in terms of spin-weighted harmonic functions,

$$
\begin{aligned}
T(\hat{\mathbf{n}}) &= \sum_{lm} c_{T,lm} Y_{lm}(\hat{\mathbf{n}}) , \\
(Q \pm iU)(\hat{\mathbf{n}}) &= \sum_{lm} c_{\pm 2,lm \pm 2} Y_{lm}(\hat{\mathbf{n}}) ,
\end{aligned}
\tag{2}
$$

where $\hat{\mathbf{n}}$ is the unit of the propagation direction, and Y_{lm} is the spherical harmonic function. One can study the gradient-like component and curl-like component by introducing a group of linear combinations of the coefficients $c_{-2,lm}$ and $c_{+2,lm}$ as follows,

$$
\begin{aligned}
c_{E,lm} &= -\frac{1}{2}(c_{-2,lm} + c_{+2,lm}) , \\
c_{B,lm} &= -\frac{i}{2}(c_{-2,lm} - c_{+2,lm}) .
\end{aligned}
\tag{3}
$$

With the assumption of statistical isotropy, all the CMB power spectra can be defined as the following expressions,

$$
\langle c_{J',l'm'}^* c_{J,lm} \rangle = C_l^{J'J} \delta_{l'l} \delta_{m'm} ,
\tag{4}
$$

where J denotes the temperature T and the E and B modes of the polarization field respectively. To neglect the local fluctuations, the CMB photons travel

through the universe by rotating their polarizations synchronously. Then we obtain the rotation relations of the power spectra as follows,

$$
\begin{aligned}
C_l^{TT,obs} &= C_l^{TT} , &(5) \\
C_l^{TE,obs} &= C_l^{TE} \cos(2\Delta\chi) , &(6) \\
C_l^{EE,obs} &= C_l^{EE} \cos^2(2\Delta\chi) + C_l^{BB} \sin^2(2\Delta\chi) , &(7) \\
C_l^{BB,obs} &= C_l^{EE} \sin^2(2\Delta\chi) + C_l^{BB} \cos^2(2\Delta\chi) . &(8)
\end{aligned}
$$

Moreover, the TB and EB correlations do not vanish,

$$
\begin{aligned}
C_l^{TB,obs} &= C_l^{TE} \sin(2\Delta\chi) , &(9) \\
C_l^{EB,obs} &= \frac{1}{2}(C_l^{EE} - C_l^{BB}) \sin(4\Delta\chi) , &(10)
\end{aligned}
$$

and thus the polarization surface can be rotated as sketch in Figure 1. If this rotation angle was observed in experiments, it would be a big challenge to the standard model of particle physics. In next sections we will discuss how a non-vanishing rotation angle appears in a Lifshitz-type non-relativistic electrodynamics.
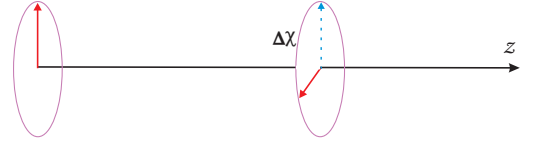

FIG. 1: A sketch plot of the generation of the rotation angle $\Delta\chi$. An electromagnetic wave is propagating along the z axis which is the horizontal one. The red arrow denotes the polarization direction. After a period of the propagation, its direction deviates from the original one.

III. BRIEF INTRODUCTION TO A LIFSHITZ-TYPE NON-RELATIVISTIC THEORY

In this section we make a brief introduction to the Lifshitz-type non-relativistic theory which is inspired by condensed matter physics[3, 20]. We work on a $d+1$ dimensional spacetime, with one time coordinate t and d spatial coordinates x^i. The theory of Lifshitz type exhibit fixed points with anisotropic scaling governed by a dynamical critical exponent z as follows,

$$
t \to b^z t , \quad x^i \to b x^i ,
\tag{11}
$$

for an arbitrary constant b. In this case, time and space have different dimensions with $[t] = -z$ and $[x^i] = -1$. Therefore, this theory breaks a Lorentz symmetry at ultraviolet limit.

Suppose we have a covariant action W in d dimensional Euclidean space for a general field $\psi^\alpha(\vec{x})$, then it gives the partition function

$$
\mathcal{Z} = \int \mathcal{D}\psi^\alpha(\vec{x}) \, e^{-W[\psi^\alpha(\vec{x})]} .
\tag{12}
$$

In the standard quantum mechanics, this partition function describes the norm of the ground-state functional, and so the vacuum wave functional takes the form

$$\Phi_g[\psi^\alpha(\vec{x})] = e^{-\frac{1}{2}W} \ . \tag{13}$$

Further, we introduce the conjugate momentum in Schrödinger-type quantum field theory with canonical commutation relation

$$[\psi^\alpha(\vec{x}), \pi^\beta(\vec{y})] = i\mathcal{G}^{\alpha\beta}\delta^3(\vec{x} - \vec{y}) \ , \tag{14}$$

where the matrix $\mathcal{G}$ is determined by the algebra in internal space of the field. In the Schrödinger representation, this commutation relation requires the operator form of the conjugate momentum to be equal to

$$\pi_\alpha(\vec{x}) = -i\frac{\delta}{\delta\psi^\alpha(\vec{x})} \ . \tag{15}$$

Using the field ψ^α and its conjugate momentum π^α, one can write down the following operator,

$$\hat{a}_\alpha(\vec{x}) \equiv i\pi_\alpha(\vec{x}) + \frac{1}{2}\frac{\delta W}{\delta\psi^\alpha(\vec{x})} \ . \tag{16}$$

If we use it to operate on the vacuum wave functional, it annihilates the ground state

$$\hat{a}_\alpha\Phi_g[\psi^\alpha(\vec{x})] = 0 \ . \tag{17}$$

Thus it has a physical interpretation as an annihilating operator, and correspondingly we can define its conjugate $\hat{a}^\dagger$ as the creating operator.

The Hamiltonian of four dimensional spacetime can be constructed by

$$H = \int dx^3 \mathcal{H}(x) \equiv \int dx^3 \frac{1}{2}\hat{a}_\alpha^\dagger \mathcal{G}^{\alpha\beta}\hat{a}_\beta \ . \tag{18}$$

Making use of the definition (16) and its conjugate, one can obtain the Hamiltonian density

$$\mathcal{H}(\vec{x}) = \frac{1}{2}\pi_\alpha \mathcal{G}^{\alpha\beta}\pi_\beta + \frac{1}{8}\frac{\delta W}{\delta\psi^\alpha}\mathcal{G}^{\alpha\beta}\frac{\delta W}{\delta\psi^\beta} \ , \tag{19}$$

Even in the classical Hamiltonian system, there is the equations of motion

$$D_t\psi^\alpha(\vec{x}) = \{\psi^\alpha(\vec{x}), H\} = \pi^\alpha(\vec{x}) \ , \tag{20}$$

where D_t is the time derivative which preserves gauge-invariance.

Eventually, a Lagrangian density can be given by

$$\begin{aligned} \mathcal{L} &= D_t\psi^\alpha\pi_\alpha - \mathcal{H} \\ &= \frac{1}{2}D_t\psi_\alpha\mathcal{G}^{\alpha\beta}D_t\psi_\beta - \frac{1}{8}\frac{\delta W}{\delta\psi^\alpha}\mathcal{G}^{\alpha\beta}\frac{\delta W}{\delta\psi^\beta} \ , \end{aligned} \tag{21}$$

and consequently we can learn that in the Lifshitz-type theory a Lagrangian density of a free field is a sum of

kinetic term that involves time derivative and a potential that is derived from the three dimensional Euclidean action. Any model obtained through the above process is said to exactly satisfy the *detailed balance condition*.

One may keep in mind that, we can add some deformations to the Lagrangian[2] if the relevant terms can keep the gauge invariance, symmetry in internal space, and renormalizability. These terms would bring a little violation of the detailed balance condition, yet is still well-defined.

IV. NON-RELATIVISTIC ELECTRODYNAMICS COUPLED WITH A SCALAR FIELD IN $d+1$ DIMENSIONAL SPACETIME

In this section we build a non-relativistic model describing a scalar field ϕ coupled with a photon gauge field A_μ in $d+1$ dimensional spacetime. Our model is required to be invariant under the following gauge transformation,

$$\delta_\lambda A_t = \dot{\lambda}/q \ , \quad \delta_\lambda A_i = \partial_i\lambda/q \ , \tag{22}$$

where q is a coupling constant with its dimension to be $[q] = 0$. Therefore, the vector sector of the Lagrangian has to be constructed by the field strengths E_i and F_{ij}, which are defined as follows,

$$E_i = \dot{A}_i - \partial_i A_t \ , \quad F_{ij} = \partial_i A_j - \partial_j A_i \ , \tag{23}$$

and the covariant derivative is given by

$$D_t = \partial_t - iqA_t \ , \quad \nabla_i = \partial_i - iqA_i \ . \tag{24}$$

Moreover, the engineering dimensions of the gauge field components at the corresponding fixed point are:

$$[A_t] = z \ , \quad [A_i] = 1 \ . \tag{25}$$

Inspired by the structure of the Lifshitz-type theory, we take the action with form of,

$$S = \int dt dx^d [(\mathcal{L}_K + \mathcal{L}_V) + \mathcal{L}_D] \ , \tag{26}$$

which is constructed by kinetic part, potentials and some possible deformations.

Using the notations of Ref. [2], we give the kinetic terms as follows,

$$\mathcal{L}_K = \frac{1}{2}(\dot{\phi})^2 + \frac{1}{2e^2}E_iE^i \ , \tag{27}$$

with e as a coupling constant with dimension $[e] = 1 + \frac{z-d}{2}$.

The potential terms are constructed from the detailed balance condition, which are given by,

$$\mathcal{L}_V = -\frac{1}{8}(\frac{\delta W}{\delta\phi})^2 - \frac{e^2}{4}\frac{\delta W}{\delta A_i}\frac{\delta W}{\delta A^i} \ , \tag{28}$$

where $W[\phi, A^i]$ is the action in d dimensional Euclidean space as introduced in previous section. The advantage of the detailed balance condition is that systems satisfy this condition have simple quantum nature relevant to that of associated theory described by the action W in lower dimensions.

Without the violation of spatial rotation, we obtain the covariant Euclidean action W with an anisotropic scaling exponent up to $z = 2$ as follows,

$$W[\phi, A_i] = \int dx^d \left[(\sigma_1 \phi \nabla^2 \phi + \sigma_n \phi^n) \right.$$
$$\left. + \frac{1}{2eg} (F_{ij} F^{ij} + m\epsilon^{ijk} A_i \partial_j A_k) \right] , \quad (29)$$

where g is another coupling constant for gauge field with dimension as $[g] = 3 - \frac{d+z}{2}$ and the last term in the integral is the only relevant deformation preserving gauge invariance, the coefficient m has dimension unity. The dimensions of the coupling constants for the scalar are $[\sigma_1] = z - 2$, and $[\sigma_n] = \frac{2-n}{2} d + \frac{n}{2} z$ respectively, where $n \geq 2$ is an integer. According to Eq. (21), we take the functional derivative of the action W_A and finally obtain the potential terms

$$\mathcal{L}_V^\phi = -\frac{1}{2} \sigma_1^2 (\nabla^2 \phi)^2 - \sigma_1 \sigma_2 \phi \nabla^2 \phi - \frac{1}{2} \sigma_2^2 \phi^2 + ... , (30)$$

$$\mathcal{L}_V^A = -\frac{1}{2g^2} (\frac{m^2}{8} F_{ij} F^{ij} + \partial_j F^{ij} \partial^k F_{ik}$$
$$+ \frac{m}{2} \epsilon^{ijk} F_{jk} \partial^l F_{li}) . \quad (31)$$

To extend, we can add some deformations to the action if the relevant terms can keep the gauge invariance and power-counting renormalizability. These terms would bring a little violation of the detailed balance condition, yet is still well-defined. For a Maxwell field and a scalar field we considered in this letter, there are three relevant terms of which the form take,

$$\mathcal{L}_D = -\frac{1}{g^2} (\kappa_0 \epsilon^{ijk} A_i F_{jk} + \kappa_1 \dot{\phi} \epsilon^{ijk} A_i F_{jk}$$
$$+ \kappa_2 \nabla_i \phi \epsilon^{ijk} A_j E_k) , \quad (32)$$

which are Chern-Simons-like terms. The operators in Eq. (32) preserves the $U(1)$ gauge transformation and also the shift symmetry $\phi \to \phi + C$ with C a constant. The requirement of the shift symmetry prohibits the large radiative contribution to the potential of the scalar field ϕ[21]. The dimensions of the coefficients are $[\kappa_0] = 3$, and $[\kappa_1] = [\kappa_2] = 3 - \frac{d+z}{2}$ respectively. In the specific case we considered, there is $d = 3$ and $z = 2$ and thus we obtain $[\kappa_1] = [\kappa_2] = 1/2$. Therefore, these deformed terms have well-defined quantum behaviors and so our model is power-counting renormalizable and ghost-free. Note that, a power-counting renormalizability does not necessarily imply the actual renormalizability. We will study this issue in details in near future.

In low energy scale, this theory flows to $z = 1$ and we expect the Lorentz invariance can be recovered as an accidental symmetry. To show that we rescale the time dimension,

$$x^0 = ct , \quad (33)$$

then we redefine the scalar field,

$$\tilde{\phi} = c^{\frac{1}{2}} \phi , \quad (34)$$

and some relevant coefficients,

$$\tilde{\kappa}_0 = \frac{8}{m^3} \kappa_0 , \; \tilde{\kappa}_1 = \frac{8c^{\frac{1}{2}}}{m} \kappa_1 , \; \tilde{\kappa}_2 = \frac{8c^{\frac{1}{2}}}{m} \kappa_2 , \quad (35)$$

by the speed of light in terms of the ultraviolet variables

$$c = \frac{em}{2g} , \quad (36)$$

and use the relativistic notation $A_\mu = (A_t/c, A_i)$.

Consequently, if we focus on the gauge field sector and its interaction with the scalar field, and work within $3+1$ dimensional spacetime, the Lagrangian becomes

$$\mathcal{L}^A = -\frac{1}{4\tilde{g}^2} \left[F_{\mu\nu} F^{\mu\nu} + \frac{\tilde{\kappa}_1}{m} \tilde{\phi} F_{\mu\nu} \tilde{F}^{\mu\nu} \right.$$
$$+ \frac{8}{m^2} \partial_j F^{ij} \partial^l F_{il} + \frac{4}{m} \epsilon^{ijk} F_{jk} \partial^l F_{il}$$
$$\left. + 2\tilde{\kappa}_0 m \epsilon^{ijk} A_i F_{jk} \right] , \quad (37)$$

where we have required $\kappa_1 = \kappa_2$ and used the signature $(-, +, +, +)$ for the metric and the effective coupling constant is

$$\tilde{g}^2 = \frac{2eg}{m} . \quad (38)$$

At the ultraviolet fixed point both e and g have dimensions of $1/2$, so $\tilde{g}$ is dimensionless. We can see that the last three terms in the Lagrangian (37) violate the Lorentz invariance explicitly while the send one violates the Lorentz invariance dynamically if $\dot{\tilde{\phi}}$ develops a non-zero time derivative which has been obtained by supposing a scalar field couples to an anomalous $B - L$ current as observed by Ref. [22]. Moreover, the possible couplings of $\tilde{\phi}$ to the standard model particles induce observable effects and have interesting implications in particle physics[21, 23–25] and cosmology[26–28] (see also [29]). However, since no definite signal of these CPT violating effects has been observed in the ground-based laboratories, we would like to see if they could leave signals on cosmological observations [6, 30, 31].

V. THE PROPAGATION OF THE PHOTON AND THE ROTATION ANGLE

In this section we will study the propagation of the photon described by the Lagrangian in the infrared limit

(37). Because A_0 is non-dynamical and this theory is gauge invariant, there are only two dynamical degrees of freedom for the photon. It is natural to choose the Coulomb gauge $\partial_i A^i = 0$ to solve the equation of motion due to the non-relativistic nature. In the absence of other source, we can set $A_0 = 0$. Besides, we require the scalar field is homogeneous and is only the function of time $\tilde{\phi} = \tilde{\phi}(t)$(we refer to [19, 32–36] for an inhomogeneous rotation).

So, the equation of motion followed from (37) is

$$\partial_0^2 A_i - \nabla^2 A_i + \frac{4}{m^2}\nabla^4 A_i + 2(\tilde{\kappa}_0 m + \frac{\tilde{\kappa}_1}{m}\partial_0\tilde{\phi})\epsilon^{ijk}\partial_j A_k$$
$$-\frac{4}{m}\epsilon^{ijk}\partial_j\nabla^2 A_k = 0 \quad (39)$$

With the ansatz of plane wave $A_i = a_i e^{ik_\rho x^\rho}$, and assume the wave is propagating along the $+z$ direction of coordinate with the wave vector $k_\rho = (\omega, \ 0, \ 0, \ k)$, we soon get $a_3 = 0$ from the gauge condition and the equations for the dynamical components:

$$(-\omega^2 + k^2 + \frac{4k^4}{m^2})a_1 - i[2k(\tilde{\kappa}_0 m + \frac{\tilde{\kappa}_1}{m}\partial_0\tilde{\phi}) + \frac{4k^3}{m}]a_2 = 0$$
$$(-\omega^2 + k^2 + \frac{4k^4}{m^2})a_2 + i[2k(\tilde{\kappa}_0 m + \frac{\tilde{\kappa}_1}{m}\partial_0\tilde{\phi}) + \frac{4k^3}{m}]a_1 = 0 \ .$$
$$(40)$$

These are algebraic equations. The nontrivial solution requires the matrix of the coefficients has null determinant. This requirement is explicitly

$$(-\omega^2 + k^2 + \frac{4k^4}{m^2})^2 = [2k(\tilde{\kappa}_0 m + \frac{\tilde{\kappa}_1}{m}\partial_0\tilde{\phi}) + \frac{4k^3}{m}]^2 \ , (41)$$

and so yields,

$$\omega_\pm^2 = k^2(1 \mp \frac{2}{m}k)^2 \mp 2k(\tilde{\kappa}_0 m + \frac{\tilde{\kappa}_1}{m}\partial_0\tilde{\phi}) \ . \qquad (42)$$

This modified dispersion relation leads to energy-dependent group velocities and will induce the time-delay of photons[37, 38], which might be an observable effect in Gamma ray burst experiments[39].

We can see from above equation that the dispersion relations for left- and right-handed circular polarized components of photons are different. The polarization angle of the linear polarized photon will be rotated by an angle $\Delta\chi$ after a period of propagation[7, 13].

In the Friedmann-Robertson-Walker (FRW) universe, the rotation angle at high frequency regime (k_{ph} is quite large) is given by

$$\Delta\chi_{UV} = \int_{z_i}^0 (\omega_- - \omega_+)dt(z')$$
$$\simeq -\frac{4}{m}\int_{z_i}^0 \frac{k_{ph}^2(z')dz'}{H(z')(1+z')} \ , \qquad (43)$$

where $k_{ph}(z') = (1+z')k_c$ is a physical frequency and z_i is the redshift when the photons were emitted. For the

CMB, the redshift is about 1100. However, if k_{ph} has already lied in the low frequency regime when the photons start propagations, the rotation angle can be derived by making use of geometric optical approximation as generally discussed in [19],

$$\Delta\chi_{IR} \simeq \int_{z_i}^0 2(\tilde{\kappa}_0 m + \frac{\tilde{\kappa}_1}{m}\partial_0\tilde{\phi})dt(z')$$
$$\simeq \int_{z_i}^0 \frac{2(\tilde{\kappa}_0 m + \frac{\tilde{\kappa}_1}{m}\partial_0\tilde{\phi})}{H(z')(1+z')}dz' \ . \qquad (44)$$

This result is similar to the case of cosmological CPT violation as considered in Ref. [6], and the current CMB data mildly indicates a non-zero central value[15–18].

Notice that, this rotation angle is frequency dependent for high frequency photons, different from that of low energy photons. This frequency dependence is different from that of Faraday rotation in which the polarization of the photon is rotated when it pass through a magnetic field. In the later case, the rotation angle is inversely proportional to the square of the frequency of the photon, in the case here however it is proportional to the square of the frequency. We note that similar frequency dependent rotation angle was studied in Ref. [40] and recently in Ref. [41] from the phenomenology of quantum gravity [42].

An important issue concerns that, if our model can be detected or constrained by current and future observations. From the above results, we can see that it is directly determined by the value of the non-relativistic scale m appeared in the Lagrangian. To be precise, a general extension of Maxwell theory with various Lorentz violating terms has been studied by Kostelecky and Mewes in Refs. [18, 40]. The third and the forth terms of Eq. (37) in our model correspond to the $k^{(5)}$ and $k^{(4)}$ ones appeared in [18, 40] respectively, if we fix $\partial_\mu\tilde{\phi}$ to be constant. Therefore, by virtue of the results obtained in Refs. [18, 40], we conclude that the non-relativistic scale m approximately lies at the level of 10^{11}GeV[1]. Then we study how this parameter affect the rotation angle at high energy scale. As a first glance, we neglect the contribution of the rolling scalar ϕ and focus on the IR limit. The current CMB polarization constraints require the term $\tilde{\kappa}_0 m$ is of order $O(10^{-3}\text{Mpc}^{-1})$. In order to let B-mode signal from the UV limit be observable, the value of mass scale m has to be promoted. Specifically, we consider the case with $m = 10^{49}\text{Mpc}^{-1}(\sim 10^{11}\text{GeV})$ which also satisfies the constraints from Ref. [40] and do the numerical calculations without any approximations, as shown in Figure 2. The result shows that the rotation angle coincides with the conventional one in Eq. (44) at low energy scale, but becomes frequency-dependent and

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[1] There is roughly $m \sim k^{(4)}/k^{(5)}$ where $k^{(4)} \sim 10^{-31}$ and $k^{(4)} \sim 10^{-20}\text{GeV}^{-1}$ from [40].

negative at high energy limit. From the recent CMB polarization observations, namely QUaD data[43], a negative frequency-dependent rotation angle might be favored at high energy scale[44, 45]. It deserve a fully detailed analysis on constraining our model with current and future cosmological observations. We will work on this issue in near future studies.
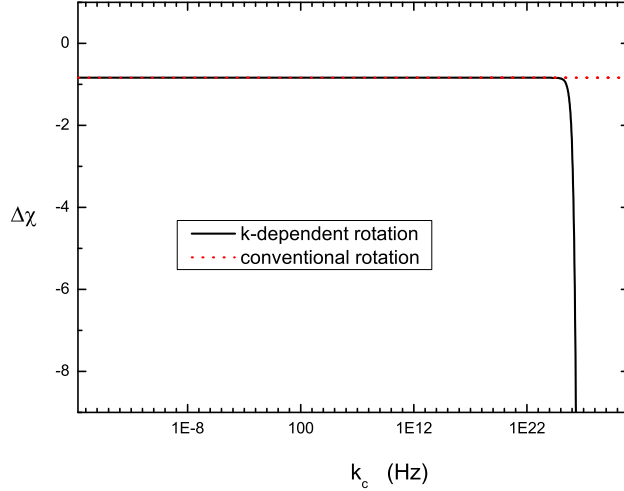

FIG. 2: Plot of the rotation angle $\Delta\chi$ as a function of the comoving wavenumber k_c. The black solid line represents the rotation angle in the non-relativistic Maxwell theory, and the red dot line represents the conventional one. In the numerical calculation, we take the following parameters $\tilde{\kappa}_0 = 10^{-52}$, $m = 10^{49} \text{Mpc}^{-1}$, and use the WMAP5 results[16] $H_0 = 70.5 km s^{-1} Mpc^{-1}$, $\Omega_{m0} = 0.2736$, $\Omega_{\Lambda 0} = 0.726$, and $w_\Lambda = -1$ is assumed.

VI. CONCLUSION AND DISCUSSIONS

The philosophy of a Lifthitz-type non-relativistic theory with quantum criticality was motivated by a quantum theory of multiple membranes designed such that the ground-state wavefunction of the membrane with compact spatial topology reproduces the partition function of the bosonic string on worldsheet[2]. Due to its advantage

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[2] Note that the scenario of multiple branes has been recently used to drive inflation as studied in Refs. [46, 47].

[3] The similar CPT breaking terms can also appear in the gravity

on power-counting renormalization and non-relativistic peculiarity, this theory has been awoken and provided plentiful phenomenons when applied in various fields and so attracted a lot of attentions. For example as pointed out by Refs. [48, 49], because of its non-relativistic feature, the gravity sector in the Lifshitz-type theory could effectively violate energy conditions and so can lead to a solution of bouncing universe [50] and the cosmological perturbations in low energy regime form a scale-invariant spectrum [51–53] with a potential red tilt[54]. However, before phenomenological applying this theory into any subjects, we ought to consider whether this theory is discernible for observations.

In this letter we have studied a non-relativistic model for a gauge field coupled with a scalar field, based on the construction of the Lifshitz-type theory. Specifically, we considered a model of electrodynamics which exhibits anisotropic scaling between time and space with the dynamical critical exponent $z = 2$. The theory is non-relativistic in essence, and hence the Lorentz invariance is violated and can only be recovered in the low energy regime as an approximate symmetry by accident. Accompanied with the violation of Lorentz invariance, the CPT symmetry is also broken due to an existence of some Chern-Simons-like terms. This CPT violation can be viewed as the Trans-Planckian physics[55, 56] and lead to the generations of TB and EB modes on CMB polarization spectra and a rotation of the polarization of the photon propagated in the universe[3], and the rotation angle is frequency dependent in UV limit while similar to the case of a dynamical violation in IR limit. This signature, if detected by future CMB observations with higher precision, would be an important clue to new physics beyond standard model.

sector of Hořava-Lifshitz theory[57] which may affect CMB tensor spectrum[58].

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